Exercise 69

After the consumption of an alcoholic beverage, the concentration of alcohol in the bloodstream (blood alcohol concentration, or BAC) surges as the alcohol is absorbed, followed by a gradual decline as the alcohol is metabolized. The function

$$C(t) = 1.35te^{-2.802t}$$

models the average BAC, measured in mg/mL, of a group of eight male subjects t hours after rapid consumption of 15 mL of ethanol (corresponding to one alcoholic drink). What is the maximum average BAC during the first 3 hours? When does it occur?

Source: Adapted from P. Wilkinson et al., "Pharmacokinetics of Ethanol after Oral Administration in the Fasting State," *Journal of Pharmacokinetics and Biopharmaceutics* 5 (1977): 207–24.

Solution

The domain of the function is $0 \le t < \infty$. Take the derivative.

$$\begin{aligned} C'(t) &= \frac{d}{dt}(1.35te^{-2.802t}) \\ &= \left[\frac{d}{dt}(1.35t)\right] e^{-2.802t} + 1.35t\left[\frac{d}{dt}(e^{-2.802t}\right] \\ &= (1.35)e^{-2.802t} + 1.35t\left[(e^{-2.802t}) \cdot \frac{d}{dt}(-2.802t)\right] \\ &= 1.35e^{-2.802t} + 1.35t[(e^{-2.802t}) \cdot (-2.802)] \\ &= 1.35e^{-2.802t}(1 - 2.802t) \end{aligned}$$

Set $C'(t) = 0$ and solve for t.

$$1.35e^{-2.802t}(1 - 2.802t) = 0$$

$$1 - 2.802t = 0$$

$$t = \frac{1}{2.802} \approx 0.356888 \text{ hours} \approx 21.4133 \text{ minutes}$$

$t = 1/2.802$ is within the interval $0 \le t < \infty$, so evaluate the function here.

$$C\left(\frac{1}{2.802}\right) = 1.35\left(\frac{1}{2.802}\right) e^{-2.802\left(\frac{1}{2.802}\right)} = \frac{1.35}{2.802e} \approx 0.177244 \ \frac{\text{mg}}{\text{mL}} \qquad \text{(absolute maximum)}$$

Evaluate the function at the endpoints.

$$C(0) = 1.35(0)e^{-2.802(0)} = 0 \qquad \text{(absolute minimum)}$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $0 \le t < \infty$.

The graph below illustrates these results.